FLOW STRUCTURE IN HYDRAULIC JUMP DURING RUN-DOWN MOTION OF SHOALING SOLITARY WAVE PROPAGATING OVER 1:3 SLOPE

Chang Lin, Guang-Wei Tseng, Ming-Jer Kao, Wei-Ing Huang, and Ching-Piao Tsai

Department of Civil Engineering, National Chung Hsing University, Taichung 40227, Taiwan.

Abstract
The characteristics of vortex structure in the hydraulic jump, which takes place during the run-down motion of a shoaling solitary wave propagating over a 1/3 slope, are investigated experimentally. A flow visualization technique and high speed particle image velocimetry (HSPIV) were used, respectively, for qualitative and quantitative illustrations. Temporal variations of the visualized serial images and the ensemble-averaged velocity fields of the hydraulic jump are demonstrated. Primary concerns are focused on formation of the separated shear layer from the 1:3 slope and on temporal and spatial variations of the vortex structure underlying the unsteady hydraulic jump.

Introduction
As first observed by Russell (1838), solitary wave propagates with stable motion over a constant water-depth channel for a long distance. They travel fairly steadily neither steepening their wave height nor widening their effective wave length, but just maintaining constant wave length and wave height. The investigation of shoaling solitary waves can be used to simulate the run-up and run-down motions as well as the shoreward inundation of a tsunami and thus enhance the ability of disaster mitigation for coastal zone.

The research for shoaling and run-up of a solitary wave over a slope has been performed by many researchers. A series of experiments was carried out by Synolakis and Skjelbreia (1993) to reveal the amplitude evolution of solitary waves propagating over a sloping bottom. Grilii et al. (1997) studied the characteristics of solitary wave shoaling and breaking on different slopes. A breaking criterion based on the value of a non-dimensional slope parameter was derived. Furthermore, Lin et al. (1999) performed numerical simulations as well as conducted laboratory experiments to study solitary wave run-up and rundown on sloping beaches. Steep- and mild-slope beaches were tested, and the consequential nonbreaking and breaking solitary waves were discussed, respectively. Velocity distribution and free-surface profiles resulted from the numerical model were verified with particle image velocimetry (PIV) measurements on the steep-slope case; however, only numerical simulations were performed on the mild-slope case. Li and Raichlen (2002) presented a numerical model for the run-up of non-breaking and breaking solitary waves propagating over a uniform slope. The process of wave breaking and ©Chang Lin, 2015
subsequent post-breaking condition in terms of bore motion were studied by Jensen et al. (2003), depicting experimental study of wave run-up of high solitary waves on a steep slope. In addition, Li and Raichlen (2003) performed experiments to explore the plunging breaking solitary waves as well as the associated bore structure and run-up. Recently, Zhang and Liu (2008) numerically investigated the dynamic process of bore formed by dam-break mechanism in a constant water depth and then traveling over a uniform slope. Sumer et al. (2011) presented two parallel experiments (with rigid-bed and sediment-bed conditions) in connection to the evolution and run-up of plunging solitary waves propagating over a sloping bottom. The experiments showed that the whole sequence of the plunging solitary wave consists of the following processes: wave shoaling and then breaking, run-up, rundown and hydraulic jump, and trailing wave. Employing PIV and wave gauges, Salevik et al. (2013) studied the characteristics of free surface elevation and velocity field of the run-up of solitary waves on a straight slope and a composite beach, respectively. Utilizing PIV and non-intrusive acoustic wave gauge, Pedersen et al. (2013), exhibited very informative data of temporal velocity profiles in the boundary layer during the run-up and rundown stages of a shoaling solitary wave propagating over a 10° sloping bed. However, the velocity measurements by PIV were only conducted at two small areas from the equilibrium shoreline, respectively. Moreover, Lin et al. (2014a) displayed the newest measurement results concerning the velocity fields of shoaling solitary wave in the prebreaking zone and near breaking point, using flow visualization technique and HSPIV. Main concerns were targeted at the near-bottom and boundary-layer flows on a 1/10 slope.

Most of previous studies of the solitary wave propagating over a slope mainly focus on variation of free surface elevation during shoaling as well as run-up and run-down processes, breaker type and breaking index, velocity field in the external flow, the maximum run-up height, and so on. The characteristics of hydraulic jump taking place during the run-down motion of a shoaling solitary wave is still very ambiguous, especially for the case of steeper slope. The systematical measurements has not yet been achieved up to now. In this study, the temporal variations of both free surface elevation and velocity fields of hydraulic jump are presented. Special attention will be paid to the separated shear layer from the sloping bottom as well as the temporal and spatial variations of vortical structure underneath the hydraulic jump.

**Experimental Setup**

The experiments were conducted in a glass-walled and glass-bottomed wave flume, having dimensions of 14.0 m long, 0.25 m wide and 0.5 m deep. The wave flume was equipped with a piston-type wave maker at one of its end, which was triggered by a servo motor; while at the other end the wave
dissipater was fixed to dampen the waves. The sloping bottom was made of acrylic with a 1:3 slope, and the toe of the slope was positioned at 9.0 m away from the wave generator. A Cartesian coordinate system is employed with the origin \((x, y) = (0, 0)\) cm at the toe of the sloping bottom; positive \(x\) and \(y\) directions are defined as the wave propagating and vertical-upward directions from the horizontal bottom of the wave flume, respectively. Figure 1(a, b) shows the schedule diagram of the wave flume and related installation of the sloping bottom, in which \(h_0\) and \(H_0\) denote the still water depth and the non-shoaling solitary wave height, respectively, and \(\eta(x, t)\) and \(h(x)\) represent the instantaneous free surface elevation and local water depth on the slope, respectively. Furthermore, \(t\) represents time and a non-dimensional time corresponding to \(t\) is defined herein as \(T = t \times (g/h_0)^{1/2}\) where \(t = 0\) (also \(T = 0\)) specifies the instant as the solitary wave crest is exactly above the toe of the slope, i.e., \(x = 0\).

![Schematic diagram of wave flume and sloping bottom](image)

Fig. 1. Schematic diagram of wave flume and sloping bottom

A flow visualization technique, using neutrally suspending particles (Titanium Dioxide powder, TiO\(_2\)), was used to qualitatively examine: (1) the occurrence of hydraulic jump during the run-down motion of a shoaling solitary wave; and (2) the underlying mechanism for the formations of separated shear layer from the sloping bottom and the induced vortical structure. Titanium Dioxide (TiO\(_2\)) powder with a mean diameter of 1.8 \(\mu\)m was chosen as the seeding particles. A laser light sheet was used to illuminate the two-dimensional motion of the tracers on a vertical plane along the longitudinal direction of the wave flume. A high-speed camera (Phantom Miro eX4 of Vision Research) was employed to capture images with a maximum framing rate of 1200 Hz and a maximum image resolution of 800 × 600 pixel.

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A HSPIV system was also utilized to obtain the two-dimensional velocity fields of hydraulic jump and the underlying vortex structure during the run-down motion of a solitary wave when flow retreated from the slope. The PIV system included an argon-ion laser as light source and a high-speed camera to capture images. Titanium Dioxide powder was also chosen as the seeding particles. Simultaneous measurements of the free surface elevations by two wave-gauges and the velocity fields by HSPIV system were carried out in the experiments. A 1,000 fps acquisition rate was used when recording images of the flow field to constitute a high time-resolved PIV algorithm. Simultaneous measurements of the free surface elevations by two wave-gauges and the velocity fields by HSPIV system at each FOV were conducted in the experiments. The wave-gauge measurement at \( x = -150.0 \, \text{cm} \) was used as a reference signal to trigger the PIV system with a preset threshold based on the incident wave condition.

**Results and Discussion**

As similarly pointed out by Lin et al. (2014b) and correspondingly shown in Fig. 2, the whole evolution process of the solitary wave propagating over the 1/3 slope can be identified for a representative case having \( H_0/h_0 = 0.363 \) (i.e., \( H_0 = 2.9 \, \text{cm} \) and \( h_0 = 8.0 \, \text{cm} \)). Namely, wave starts shoaling at \( x/h_0 = 0 \) and \( T = 0 \) and then enters the shoaling zone for \( T > 0 \). Wave does not break due to steep slope. Subsequent run-up motion takes place during \( T = 5.7 \sim 7.9 \) and run-down motion occurs for \( 7.9 < T < 14.0 \), in between the hydraulic jump and flow separation forms at \( x/h_0 = 2.50 \) and 2.25 for \( T = 10.0 \) and 10.9, respectively. Hydraulic jump takes place for \( 10.0 < T < 14.0 \), and finally the second run-up starts for \( T > 14.0 \). The maximum thickness of vortex structure during the hydraulic-jump event is about 2/3 of the instantaneous water depth.

![Fig. 2. Non-dimensional timeline showing an entire evolution of a shoaling solitary wave with \( H_0/h_0 = 0.363 \)](attachment)

It should be noted that, during the former stage of run-down, the boundary layer flows are all within the laminar regime for the present case at a 1:10 slope (Lin et al. 2014b). Nevertheless, during

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the later stage of run-down, high-velocity retreated flow takes place in the very shallow area, behaving like a supercritical flow or a gravity sheet-current flow in a steep channel (Sumer et al. 2011). On the other hand, flow with relatively low-velocity happens in the region of larger water-depth offshore side, and acts like a subcritical flow. The unsteady hydraulic jump takes place consequently due to decreasing water depth with chaotic flow at both the free surface and in the flow beneath the jump.

The visualization results of flow images over the sloping bottom near and at the flow separation point at about $x/h_0 = x_{fs}/h_0 = 2.25$ for the present case ($H_0/h_0 = 0.363$) are depicted in Fig. 3(a-h) for $T = 10.9 \sim 12.03$. Note that these images were selected from movie pictures being continuously recorded by the high speed digital camera (Phantom V5.1 of Vision Research) with a frame rate of 1000 Hz. The practical reason for using this camera is to take photos with higher framing rate but with monochrome fidelity, aiming to observe the evolution of vortical structure underneath the hydraulic jump with higher temporal and spatial resolutions. As evidenced in Fig. 3(a), the beginning of flow separation due to the pressure differences both along and normal to the sloping bottom is seen clearly at about $x_{fs}/h_0 = 2.25$ for $T = T_{fs} = 10.9$. After that, the thickness of shear layer grows up and first vortex forms underlying the shear layer [see Fig. 3(b,c)] for $T = 11.10$ and 11.78, respectively. Subsequent evolution of the first vortex structure, characterized by the increase of vortex size and vortex strength between the shear layer and the sloping bottom, are observed clearly in Fig. 3(d-f) for $T = 11.84, 11.88, \text{ and } 11.95$, respectively. A high speed flow between the free surface and the first vortex structure, behaving like a jet over the first vortex which is followed by a smaller second vortex, are clearly observed. Further, the evolution of the third vortex closely following the first vortex with the smaller second vortex in between can be evidenced in Fig. 3(f,g). However, the size and strength of the second vortex are less than those of the first vortex. Note that, based on the fact mentioned above, the maximum thickness of vortex structure during the unsteady hydraulic-jump event may reach about 2/3 of the instantaneous water depth for the present case.

Fig. 4(a-d) presents the corresponding ensemble-averaged velocity fields in the run-down motion of this solitary wave (having $H_0/h_0 = 0.363$) measured by HSPIV in the range of $12.7 \text{ cm} < x < 19.0 \text{ cm}$ for $T = 9.26, 10.24, 10.89, 11.72, 11.90$ and 12.09, respectively. The corresponding Froude number, based on the local water depth and the depth-averaged velocity, is one-by-one identified and shown in Fig. 5 The deeper the water depth the smaller the mean velocity in the observed section. The corresponding Froude number at every section is smaller than 1.0. This situation clearly shows the flow is of subcritical. However, it is surprisingly found that subcritical flow takes place for deeper water region at $x < 20.0 \text{ cm}$ (i.e., $Fr < 0.76$); a critical flow nearly happens at $x = 20.0 \text{ cm}$ (i.e., $Fr = 0.993$) and supercritical flow occurs for shallower water region at $x > 20.0 \text{ cm}$ (i.e., $Fr > 1.25$), demonstrating the start of the hydraulic jump at about $x = x_{hj} = 20.0 \text{ cm}$ and $T_{hj} = 10.0$. ©Chang Lin, 2015
Fig. 3(a-h). The visualization results of flow images over the sloping bottom near and at the flow separation point for $T = 10.89 \sim 12.03$. 

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A similar situation for $T = 10.3$ and $x = 20.0 \text{ cm}$ can be also evidenced because subcritical and supercritical flows take place in the deeper and shallower water regions for $x < 20.0 \text{ cm}$ and $x > 20.0 \text{ cm}$ and critical flow occurs at about $x = 20.0 \text{ cm}$, respectively. The inciency of flow separation is at $x_{fs} = 19.0 \text{ cm}$ and $T_{fs} = 10.9$, in which the former is in accordance with the initial position of the unsteady hydraulic jump; and the latter is a little later than that of the hydraulic jump (i.e., at $T_{hj} = 10.0$). Note that the corresponding Froude numbers for the incipient occurrences of hydraulic jump and flow separation are the same and equal to unity, i.e., $Fr = 1.0$.

![Fig. 4(a-f). The ensemble-averaged velocity fields in the run-down motion of shoaling solitary wave in the range of 12.7 cm < x < 19.0 cm](image-url)
Fig. 5. The evolution of Froude number, based on the local water depth and the depth-averaged velocity, measured at different locations

Conclusions

The characteristics of vortical flow fields underneath the hydraulic jump taking place during the run-down motion of a shoaling solitary wave propagating over a 1/3 slope have been studied experimentally. Key findings of this study are the occurrence condition for the unsteady hydraulic jump, the formation of the separated shear layer from the sloping bottom, and the representative futures of the temporal and spatial variations of vortical structure underlying the unsteady hydraulic jump. The corresponding Froude numbers for the occurrences of hydraulic jump and flow separation are equal to unity.

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