LAGRANGIAN-BASED VISUALIZATION OF STAGNATION VOXEL PAIR
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Introduction
It is well known that the stagnation flow in front of a bluff body can be significantly affected by
free-stream vortical disturbance\[1, 2\], which has practical use in convective heat transfer
enhancement. Recent experiment\[3\] has shown that the wake of an upstream-positioned thin
interference wire can induce a pair of counter-rotating vortices in the stagnation region of a circular
disk, which has a size one order larger than that of the wire. This vortex pair grows steadily around
the centre of the disk till an elliptic instability occurs on the vortex pair, leading to a quick shedding
from either direction.

In\[3\], the dynamic process of vortex formation/growth/shedding was shown by means of
vorticity contour plots obtained from 2D PIV measurement. From the sense of vortex identification,
vorticity is not a good choice of vortex indicator because of two aspects: 1) the ambiguity between
vortical structure and background shearing and 2) the manually selected threshold to define the
vortex boundary.

The widely-used vortex identification schemes include $Q$-criteria\[4\], $\lambda_2$-criteria\[5\], $\Delta$-criteria\[6\]
and $\lambda_{ci}$-criteria\[7\]. All these schemes are based on Eulerian system, using certain characteristics of
invariant of velocity gradient tensor $\nabla \mathbf{v}$ to depict vortex. In order to compute eigenvalues of $\nabla \mathbf{v}$,
three-dimensional instant velocity field with adequate spatial resolution is required, which is
difficult for present experimental techniques. Another kind of schemes belongs to Lagrangian
system, time-integrated properties of fluid particle trajectories are used to identify vortical
structures. One of the typical Lagrangian-based schemes is Finite-Time Lyapunov Exponents
(FTLE) method\[8, 9\]. This method is based on the concept that vortex boundary can be regarded as
the hyperbolic material surface which distinguishes fluids inside the vortex from its surrounding
background, and thus uses time-integrated divergence/convergence rate of neighbouring fluid
particle trajectories as the indicator. Thus it is a more direct method for quantitative visualization,
and has numerical stability for noise contamination in experiment measurement. Here we use this
Lagrangian-based vortex identification method, Finite Time Lyapunov Exponents ($FTLE$), to
visualize the vortex dynamics from PIV-measured velocity field.

Experiment setup
The experiment is conducted in a low-speed water tunnel with the free-stream velocity $U_\infty=66$
mm/s and free-stream turbulence intensity $Tu<0.8\%U_\infty$. As shown in Fig.1, a circular disk with
diameter of $D=60$ mm is placed perpendicular to the free stream to form stagnation flow. A Nylon
fishing line with diameter of $d=0.33$ mm and length of 200 mm is stretched 60 mm upstream of the
disk. The Reynolds number of the fishing line is $Re_d=22$ so that vorticity non-uniformity, instead of
periodical Kármán vortex street, dominates the convective wake. Time-resolved 2D PIV
measurement with sampling frequency of 140Hz is taken at the symmetric $(x, y)$ plane $(z=0$ mm).

From Lagrangian viewpoint, vortex boundary can be regarded as the hyperbolic material surface,
which distinguishes fluids inside the vortex from the surrounding background. Considering a pair of
particles with an infinite distance which are separated by the vortex boundary, due to the
rotating/spiraling motion of the vortex, these two particles either depart from or approach each other within a short time interval. Green et al.[8] and Haller et al.[10] mathematically proved that particle pairs located at the vortex boundary own the maximum stretching/folding rate. In this sense, the vortex boundary can be objectively defined as the ridges in FTLE field which are gradient lines of FTLE field transverse to the direction of minimum curvature.

![Fig.1 Sketch of experimental set-up to show the vortal structure in an axi-symmetric stagnation flow[3]](image)

Detailed description of the experiment can be found in[11]. In short, FTLE field can be computed by: (i) evenly distributing imaginary particles with sufficient number in the flow field at initial time \( t_0 \); (ii) tracking the convection of both the particle located at the specified space point \( X_{i,j} \) and its nearest neighbors with initial distance small enough over time interval \( T \); (iii) calculating the relative distance \( D(T; t_0; X_{i,j}) \) between \( X_{i,j} \) and its neighbours at the time of \( t_0+T \). To advect particles in the flow field, Eulerian velocity field with sufficient temporal resolution is needed. Fig.2 gives the illustration of advecting particle groups. The relative distance between the central particle \( X_{i,j} \) and its neighbours is defined as

\[
D^2(T; t_0) = \frac{1}{N} \sum_{m=\pm 1, n=\pm 1} |X_{i,j}(t_0 + T) - X_{i+m,j+n}(t_0 + T)|^2
\]

Haller et al.[12] derived that the expansion rate \( \sigma_T(x_0, t_0) \) of the relative distance between neighbouring trajectories is equal to the square of the largest singular value of the deformation gradient \( \frac{\partial x(t_0 + T, x_0, t_0)}{\partial x_0} \):

\[
\sigma_T(x_0, t_0) = \lambda_{\text{max}} \left( \left[ \frac{\partial x(t_0 + T, x_0, t_0)}{\partial x_0} \right]^T \left[ \frac{\partial x(t_0 + T, x_0, t_0)}{\partial x_0} \right] \right)^{1/2}
\]

where \( x(t_0 + T, x_0, t_0) \) is the spatial position of the particle initiated at \( (x_0, t_0) \) after advecting over time interval \( T \). The corresponding FTLE at point \( x_0 \) is then computed as

\[
FTLE_T(x_0, t_0) = \frac{1}{2T} \log \sigma_T(x_0, t_0)
\]

The ridges of the FTLE field thus indicate attracting material lines with maximum convergence rate, analogous to the accumulation of dye/hydrogen bubbles collected by vortical structures in flow visualization.
Results and discussion

Fig. 3 gives a sequence of flow visualized pictures by Hydrogen bubble visualization, showing a development of counter-rotating pair of vortices in the frontal stagnation region of a circular plate. The vortex boundary is clearly visualized. Fig. 4 shows one period of vortex pair evolution process by using both FTLE visualization and traditional vorticity-streamline plots. It is clearly shown by FTLE method (the first row in Fig. 4) that the vortex pair formation is due to the accumulation of upstream wake vorticity with the help of strong shear in the stagnation region. The visualized pattern is very similar to that in Fig. 3. The boundary of the clock-wise rotating vortex (the upper one in the vortex pair) becomes corrugated at $t + 4\Delta t$, implying the onset of elliptic instability. Moreover, the center of this clockwise rotating vortex is closer to the wall than that of the anti-clockwise rotating vortex (the bottom one), which results in a 'kick-away' movement of the vortex pair towards upper direction during $t + 4\Delta t \sim t + 7\Delta t$. Comparing to the FTLE visualization, the vorticity-streamline plots (the second row in Fig. 4) depicts the growth and shedding process of the vortex pair in the similar sense, however, it cannot provide abundant information of the elliptic instability which initially acts on the vortex boundary.

Fig. 5 shows another period of vortex pair evolution within which the vortex pair finally sheds towards the bottom direction. The corrugation and deformation of the vortex boundary during the shedding process can be also well visualized by means of FTLE method.

In conclusion, the dynamical evolution of the stagnation vortex pair induced by the interference wire wake is visualized by a Lagrangian-based FTLE method. This method excels traditional Eulerian vortex identification method in depicting the vortex boundary. Therefore, the elliptic instability of the stagnation vortex pair can be further investigated in the Lagrangian perspective.
Fig. 3 A sequence of flow visualized pictures to show the development of a vortex pair shedding towards upward direction. The time interval between the consecutive pictures is $\Delta t^* = 0.6$.

Fig. 4 Visualization of vortex pair formation, growth and shedding towards upper direction. Contour maps in the first row show the scalar field of FTLE, and contour maps in the second row indicate the vorticity distribution in combination with instantaneous streamline (the same for Fig. 2), $\Delta t=0.829$ s.
**Fig. 5** Visualization of vortex pair formation, growth and shedding towards bottom direction, $\Delta t=0.829$ s.

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**REFERENCES**


