THE VERIFICATION AND EXPERIMENT OF OPTICAL FLOW METHOD

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Introduction. The quantitative measurement of flow velocity field possesses fundamental and key importance for understanding complex flow phenomenon and physical nature in the research of aerodynamics, at the same time, the demand also stimulates the development of aerodynamic test technology level. Traditional flow measurement methods mostly adopt single point measurement or invasive measurement techniques [1], cannot meet the needs of complex flow situation and global measurement.

In recent decades, depending on particle image and cross correlation method to acquire velocity vector field, the Particle Image Velocimetry had achieved considerable development and hold application advantages of non-invasive measurement, global measurement and high accuracy [2]. The velocity vector obtained by PIV is the average velocity of interpretation window, so the variation of velocity field in the interpretation window is ignored. During the exposure interval, the displacement of particle, which follows the nyquist criterion, should be less than half of the size of interpretation window so that the spatial resolution of PIV is limited by certain restrictions. The accuracy of velocity vector field obtained by PIV is also restricted in the region with large velocity gradient.

Images of object recorded by camera are essentially distribution of reflected light from object. When human’s eyes observe moving object, the motion scene of the object will show a series of continuous varying images passing through the retina (i.e. the image plane), just like a stream of light, which is called “optical flow”, passing through the retina. Under the condition of illumination, the surface grayscale of an object will presents certain spatial distribution, which is called grayscale pattern, and the So-called “optical flow” refers to the motion of the grayscale pattern. The optical flow image represents the spatial distribution of some kind of physical quantity, such as temperature, concentration, density or the grayscale space distribution. The optical flow coming from the movement between observer and observed object can reveal the motion information of the observed object. Particle images record reflected light from particle swarm and show particle concentration distribution, so velocity vector field by PIV method is essentially velocity vector field of “optical flow”.

Horn and Schunck creatively studied optical flow in 1980s [3]. Horn et al thought that the optical flow field caused by moving object should be continuous and smooth, namely the grayscale variation projected on the image should be smooth because velocities of adjacent points on the same object should be similar. Based on this idea, additional velocity smoothness constraint, i.e. global smoothness constraint can be added to optical flow field, so the calculation problem of optical flow field could be translated into the calculus problem of variation and the motion estimation of pixel scale would be obtained. Subsequently, the optical flow technology attracted attention of researchers [4–6], some calculating methods, such as image correlation method, energy method [7], phase method [8] and neutral dynamics method, are proposed. Aiming at the difficult mentioned by TianShu Liu [9] and faced in differential method proposed by Horn when calculating the larger pixel displacement, Corpetti put forward using optical flow in the form of integral equation instead of the differential form of variational equation [10], Ruhnau etc applied Gaussian filter to create the multi-resolution strategy of binary image pyramid [11]. These algorithms have their own advantages, but due to a certain scope of application and algorithm complexity, so it is unrealistic for these algorithms to get a wide range of applications relative to the differential method.
In the paper, optical flow calculation method was introduced and optical flow test program was programmed. Through artificial displacement image, the program was verified and analyzed to obtain a reasonable measurement range and the optimal weighted value by which to guide how to gain optical flow velocity field of high precision in practical application. A verification experiment was carried out in a small wind tunnel, by tracer particle as measurement carrier, and the velocity vector field with a resolution of pixel scale was calculated out. The objective of this paper is to develop full flow field measurement technology based on optical flow, improve the spatial resolution and avoid the velocity gradient effect. Through these studies, it is expected that optical flow can contribute to understanding the dynamics space structure of complex flow and acquiring more dynamics information.

**The solution of Optical flow velocity field.** The differential method is the base of the solution of Optical flow velocity field. Based on optical flow constraint equation combined with smoothness constraint condition and spatial and temporal gradients of grayscale, the movement information of each point would be extracted from optical flow field.

In the optical flow constraint equation, grayscale \( I \) satisfies the following equation:

\[
I(x, y, t) = I(x + \Delta x, y + \Delta y, t + \Delta t)
\]  

(1)

Here \( I(x, y, t) \) is the image grayscale on time \( t \) and location \( (x, y) \), and \( I(x + \Delta x, y + \Delta y, t + \Delta t) \) is the image grayscale on time \( t + \Delta t \) and location \( (x + \Delta x, y + \Delta y) \). Let \( u=\Delta x/\Delta t \), \( v=\Delta y/\Delta t \) and ignoring higher order terms, optical flow constraint equation can be obtained by Taylor expansion.

\[
I_xu+I_yv+I_t=0
\]  

(2)

\( I_x, I_y, I_t \) are two-dimensional spatial and temporal gradients of grayscale. Obviously, this equation is an indeterminate equation with two unknowns \( u, v \). So the additional constraint equation needs to be introduced here. Each point of the image grayscale pattern could not move independently but correlatively, and velocities on adjacent positions should be continuous, i.e. the velocity smoothness constraint of image grayscale pattern. One of expressions of additional velocity smoothness constraint is minimizing the quadratic sum of the optical flow velocity gradients \((\partial u/\partial x)^2 + (\partial u/\partial y)^2 \) and \((\partial v/\partial x)^2 + (\partial v/\partial y)^2 \). Let the criterion error of optical flow constraint equation is:

\[
\xi_i = I_xu+I_yv+I_t
\]  

(3)

The criterion error of velocity smoothness constraint is:

\[
\xi_i^2 = (\frac{\partial u}{\partial x})^2 + (\frac{\partial v}{\partial y})^2 + (\frac{\partial u}{\partial y})^2 + (\frac{\partial v}{\partial x})^2
\]  

(4)

As the effect of experimental noise and hardware noise and discrete error, the image grayscale would deviate from the basic constraint condition, and it is difficult for \( \xi_i \) to be constantly equal to zero. So it is essential to find an appropriate weighted factor \( \beta \) to ensure the integration minimization of two criterion errors on two-dimensional space:

\[
\xi^2 = \int \int (\xi_i^2 + \beta^2 \xi_i^2) dx dy
\]  

(5)

Through variational method and minimization, Euler characteristic equations of each dependent variable could be established.

\[
I_x^2u + I_y^2v = \beta^2 \nabla^2 u - I_x, I_t
\]  

(6)

\[
I_xI_y + I_t^2v = \beta^2 \nabla^2 v - I_y, I_t
\]  

(7)
Here $V^2$ is Laplace operator. After two Euler characteristic equations had been established, two spatial and temporal discrete schemes of grayscale would be obtained by spatial and temporal discretization. From discrete schemes, two iterative discretization schemes of two velocity components could be established, and the movement information $(u, v)$ would be extracted from optical flow field [12].

**The verification of optical flow algorithm program.** The optical flow algorithm program, which would be used to measure the velocity of wind tunnel experiment, was completed based on differential method. In order to verify the algorithm's accuracy, several grayscale images shifted by given displacement were used for image processing by the optical flow algorithm program, the result of which will be compared with the standard displacement. In the actual calculation, 1 unit time gradient was adopted so that the value of the velocity would directly reflect the pixel displacement and inspect the image pixel displacement's influence to the algorithm accuracy.

The first image pair which are shown in Figure 1a are the original and its horizontal shifting image with 1 pixel to the right ($\Delta x=1$ Pixel), and the velocity vector distribution and the streamlines gained from this image pair are shown in Figure 1b, which indicate that the velocity field calculated from the optical flow algorithm is very uniform in this case.

![Fig. 1. Original and its horizontal shifting image (a) and velocity vector distribution and the streamlines (b) and horizontal and vertical velocity distribution](image)

The velocity distribution located on the horizontal inspection line and the vertical inspection line shown in Figure 1a is analyzed in Figure 1c, which indicates that velocity field calculated from the optical flow algorithm has almost no deviation from the standard displacement.

The second image pair which are shown in Figure 2a are the original and its slanting shifting image with 1 pixel to the right ($\Delta x=1$ Pixel) and 1 pixel to the top ($\Delta y=1$ Pixel), and the velocity vector distribution and the streamlines gained from this image pair are shown in Figure 2b, which indicate that the velocity vector distribution is uniform and the flow field is smooth, but partial velocity deviation has emerged in this case.

By comparing the horizontal velocity distribution and the vertical velocity distribution with the standard (Figure 2c), it can be concluded that the velocity value has a high accuracy but a certain amount of jitter deviation in this case. This deviation could be related to speed deviation’s synthesis in both directions and it also indicates that the error of optical flow algorithm will increase with raising pixel displacement between two adjacent images.
Fig. 2. Original and its slanting shifting image (a) and velocity vector distribution and the streamlines (b) and horizontal and vertical velocity distribution

The third image pair which are shown in Figure 3a are the original and its rotting image with 1 degree around the image center, and the velocity vector distribution and the streamlines gained from this image pair are shown in Figure 3b, which indicate that the velocity vector distribution is uniform and the flow field is smooth in the central part, but larger velocity deviation has emerged in the edge part in this case.

Fig. 3. Original and its rotating image (a) and velocity vector distribution and the streamlines (b) and horizontal and vertical velocity distribution (c)
By comparing the horizontal velocity distribution and the vertical velocity distribution with the standard (Figure 3c), it can be concluded that the velocity value has a high accuracy in the part of where the pixel displacement is below 2 pixels but an obvious deviation in the part of where the pixel displacement is over 2 pixels. This explains that why the velocity vector distribution is uniform and the flow field is smooth in the central part, but with larger deviation in the edge part, and also largely guides how to ensure access to high accuracy optical flow calculation result in the actual measurement applications.

The optimal estimation of weighted factor As a weighted value, \( \beta^2 \) reflect the relative influence of the experimental and numerical discrete noise. In actual application, by adjusting the proportion of smoothness constraint, different smoothness level’s velocity vector field could be obtained. Below, with the rotating simulation as an example, vertical velocity distributions of different \( \beta^2 \) were compared with standard velocity distribution (Figure 4).

![Fig. 4. Velocity distributions of different \( \beta^2 \) compared with the standard velocity distribution. \( \beta^2=1(a) \) and \( \beta^2=4(b) \) and \( \beta^2=7(c) \) and \( \beta^2=10(d) \) and \( \beta^2=13(e) \)](image)

From Figure 4, it indicates that, along with the increase of \( \beta^2 \), the horizontal velocity distribution could better match the standard velocity distribution. It suggest that the appropriate increase of \( \beta^2 \) will appropriately improve the accuracy of computation.

Verification Experiment of Optical Flow Velocity Field Measurement. The preliminary calculation program had been compiled according to the optical flow algorithm, and then was verified by the vortex structure observation experimental of backward facing step interval flow field.

The difficulty in this experiment is to guarantee the experiment conditions satisfying the constraint equation of optical flow velocimetry. For effective calculation of optical flow velocity field, this algorithm requires identical illumination condition for each image of continuous shooting and tiny displacement of corresponding pixels on adjacent images.
The experiment was conducted in a small low-speed wind tunnel system which was set up temporarily in the laboratory. The whole system was composed of wind tunnel and flow control sub system, tracer particles generation and seeding sub system, laser illumination and optical path sub system, image acquisition sub system and computer processing sub system. The experiment schematic diagram is shown in Figure 5.

![Experiment schematic diagram](image)

Fig. 5. Experiment schematic diagram (a) and optics arrangement and test section (b) and seeding rake (c) and tracer generator (d) and experiment image (e)

The flow speed of wind tunnel test section is controlled by a controllable turbo fan motor and within the range of 1m/s to 20 m/s by regulating the frequency converter. The wind tunnel inhaled air flow mixed with tracer particles coming from three seeding tubes, and after passing through the cellular, the air and the tracer particles mixed uniformly. An Argon ion continuous laser, beam expander lens and cylindrical lens were applied in this experiment to achieve identical illumination condition. A high-speed camera with maximum frequency of 1500HZ was used to capture images and ensure tiny displacement at scale of pixel between adjacent images. A calibration plate should be placed in the experiment area, and CCD camera would record the image of calibration plate to determine the actual measurement area size.

**Analysis of optical flow of particle image and normalization of particle image.** The image of backward facing step internal flow field captured by the high-speed camera is shown in Figure 6a, and the dashed box in the image is the computational domain. A large number of continuously captured images recorded the information of continuously varying optical flow field. To verify the effectiveness of the optical flow algorithm program for application in particle images, one particle image, as shown in Figure 6b, was rotted with one degree, and the velocity vector distribution and the streamlines gained from this image pair is shown in Figure 7a. Also the comparison between computation result and standard result is shown in Figure 7b, and the relative error and the absolute error are shown in Figure 7c.

From Figure 7, it can be known that the velocity value could well match standard velocity value when the displacement was less than one certain value. The absolute error is basically less than 0.05 pixels, but because of little absolute value of displacement, the relative error is bigger for the rotation center region. When the displacement is about one unit pixel, absolute error and relative error are minimal and reach the level of one percentage. In the neighbourhood of unit
pixel displacement for about one half pixels, the absolute accuracy and relative accuracy of velocity are high.

![Flow field image of Backward facing step and computation domain](image1)

**Fig. 6.** Flow field image of Backward facing step and computation domain (a) and original particle image and its rotating image (b)

![Velocity vector and the streamlines obtained by the optical flow algorithm](image2)

**Fig. 7.** Velocity vector and the streamlines obtained by the optical flow algorithm (a) and horizontal and vertical velocity distribution compared with the standard velocity distribution (b) and absolute error and relative error of the velocity on both of the examination lines

Due to the expanding angle and energy’s Gaussian distribution during the propagation process of laser sheet, the width of laser sheet is getting bigger all the time and energy distribution is not uniform and different along with the temporal and spatial variation. In addition, as a result of manufacturing processes and abrasion of plexiglass of experiment section, there are many tiny air bubbles in plexiglass and slight scratches on the surface. When laser sheet pass through plexiglass and meet air bubbles and slight scratches, the energy of laser sheet will be partly cut off, and the laser sheet will contain some stripes. All these will lead that the energy distribution of laser sheet is discontinuous and the greyscale field contain some stripes.

Because of reasons mentioned above, the greyscale field captured by high-speed camera can’t reflect the true scalar field (concentration distribution) and must be calibrated. One image should be captured as calibration image in the environment of uniform scalar field. While the greyscale field of calibration image is discontinuous, but it records optics conditions of each points. The greyscale filed $I$ captured in experiment should be divided by the greyscale field $\hat{I}$ of calibration image, and the calibrated greyscale field i.e. the relative greyscale field $I'$ would be obtained, as shown in below:

$$I' = I / \hat{I}$$  \hspace{1cm} (8)

If further considering noise, the noise field $I_{\text{noise}}$ should be captured, and the relative greyscale field $I'$ could be further expressed as below:

$$I' = (I - I_{\text{noise}})(\hat{I} - I_{\text{noise}})$$  \hspace{1cm} (8)
For achieving a better result, different weighted values were chosen to calculate the correlation coefficient $r$ between optical flow result and standard result (Figure 8). From the curve, the correlation coefficient $r$ rapidly increases when $1 \leq \beta^2 \leq 8$; the correlation coefficient $r$ gently increases when $8 \leq \beta^2 \leq 16$; the correlation coefficient $r$ gradually change when $\beta^2 \geq 16$; the correlation coefficient $r$ reach the maximum value when $\beta^2 = 20$. The choice of $\beta$ should be further researched.

![Fig. 8. Correlation coefficient curve](image)

**Experiment result and analysis.** The experiment result is shown in Figure 9a, the maximum displacement is 2.2 pixels and the computation result is in the confidence interval. Because of low absolute velocity in the measurement region, the flow field structure is not typical. So the experiment result calculated by optical flow algorithm program was compared with PIV result (Figure 9b). Streamline images of optical flow result and PIV result are shown in Figure 10. From Figure 9 and Figure 10, it can conclude that, under the condition of same pixel resolution, the optical flow result is consistent with that of PIV at velocity magnitude, direction and velocity contour and there is no distinct difference in the structure and location of large scale vortex, but the optical flow result is smoother for velocity contour.

![Fig. 9. Comparison of Flow field result, optical flow result (a) and PIV result (b)](image)

![Fig. 10. Comparison of Streamline result, PIV result (a) and optical flow result (b)](image)
From Figure 10, it also can conclude that, under the condition of same pixel resolution, the structure of large scale vortex can be viewed in PIV result, but the structure of micro scale vortex cannot be clearly viewed while can be clearly viewed in optical flow result. For further analyzing the difference between optical flow and PIV, the micro scale vortex is amplified in Figure 11. The optical flow result, in which there is a velocity vector on each pixel, can show a more detailed flow field structure. The PIV result with a resolution of 8×8 pixel has an obstacle on viewing fine flow field structure.

**Conclusion and prospect.** Through the analysis of optical flow algorithm, the experiment requirement of optical flow velocimetry was studied and the test platform was successfully set up. By comparison between optical flow result and PIV result, it can be concluded that the optical flow algorithm could obtain a smoother velocity field than PIV in the conditions of pixel scale and is more suitable for velocity field measurement of complex flow.

**Acknowledgement.** This work was supported by National Key Basic Research Program of China (2014CB744805)

**REFERENCES**