Abstract. The trailing vortices which reflect the lift as well as drag characteristics of the aircraft could be optimized for better aircraft performance characteristics and reduction of environmental impact as well as hazardous interference to other flight vehicles and/or objects. To that end the present work is addressed to the analysis and visualization studies of near field trailing vortices to obtain insight on the rolling moments induced by them in view of passive wake alleviation. The analysis and simulation studies are based on a basic and simple approach using desktop computer and fundamental principles for educational purposes. Within such frame of thought, various vortex Aircraft Trailing Vortices Vortex Models are reviewed and analysed in view of their potential Wake Hazard Alleviation, by simulating the rolling moment induced. The simulation of various vortex model chosen are judged by their expected characteristics that may reproduce actual state of affairs.

Introduction. The contribution of vortices to lift and lift enhancement bring our attention to the efforts for induced drag minimization and accordingly, trailing vortices alleviation. This subject has received much attention and has prompted increased multidisciplinary research effort. Lift is associated with bound vortex. Annular Wing will not shed trailing vortices, but favorable Lift and Lift to Drag Ratio compared to conventional wing will be of interest. Spiroid wing may offer a compromise between the two paradigms, as one of the effort to enhance lift and minimize trailing vortices effects. These remarks may illustrate the significant key to increasing lift to drag ratio, thus minimize trailing vortices induced downwash, which may be the redistribution of trailing vortices intensity in the Trefftz plane.

The prediction and control of trailing vortices (Figure 1) presents both a technological challenge of importance in air traffic control and provide a myriad of instructive phenomena in fluid mechanics. The mechanism underlying these phenomena, and its control for various applications, in particular for flight safety, has been the subject of intensive multidisciplinary field of research (Jacquin, [1]).

The state of affairs of wingtip vortices which form the major component of wake turbulence is illustrated in Figure 2, indicating the flow fields of vortices which possess intense swirl velocities and complex time-dependent behavior. Wingtip vortices are associated with induced drag, which is a direct consequence of the generation of lift due to viscous effect.

Principles Of Trailing Vortex Dynamics. Breitsamter [5], in his overview on wake vortex research including early studies, model and flight tests, numerical investigations and fundamental physical aspects and alleviation strategies identifies characteristic quantities for wake vortex analysis including typical length and time scales as well as turbulence quantities. From his analysis on detailed results on the near field wake vortex properties associated with large transport aircraft in approach configuration, and in the context of the far field development, instability mechanisms are elucidated along with their relevance for wake vortex decay as illustrated in Figure 3.

Judging from its downstream development, Breitsamter [5] divided a vortex wake into four regions, as depicted in Figure 3.

(i) The near-field, \( x/b \leq 0.5 \), \( (x/l_\mu \leq O(1)) \), which is characterized by the formation of highly concentrated vortices shed at all surface discontinuities.
(ii) The extended near-field, $0.5 < x/b \leq 10$, where the wake roll-up process and the merging of dominant vortices (e.g. shed at flap edge, wingtip, etc.) take place, which further downstream gradually becomes two counter-rotating vortices.

![Figure 1](image1.png)  
**Figure 1:** (a) Picture of trailing vortices, (b) Hypothetical concept of Aircraft with Annular Wing, (c) Conception of Aircraft with Spiroid winglet, (d) Impression of the trailing vortex system shed by an aircraft due to wing and horizontal tail (from Boeing Aero Magazine [2] and Crouch & Spalart patented document [3]).

![Figure 2](image2.png)  
**Figure 2:** (a) Wake vortex evolution and roll-up process, originated from surface discontinuities on the wing Huenecke [4], (b) Non-dimensional axial vorticity distribution at $x^*=0.37$ for reference configuration 1 (E403) and shedding location of dominant near field vortices, indicating the relationship between qualitative circulation distribution and trailing vortices development (Breitsamter [5]).

![Figure 3](image3.png)  
**Figure 3:** Stages of wake vortex lifespan, showing: initial vortex perturbations, the intermediate development, and the far-field result of active control using conventional aircraft control surfaces adapted from Breitsamter [5].

(iii) The mid- and far-field, $10 < x/b \leq 100$, where the wake is descending in the atmosphere and linear instabilities emerge.

(iv) The dispersion region, $x/b > \sim 100$, where fully developed instabilities cause a strong interaction between the two vortices until they collapse.

Addition of turbulence to the vortex wakes of aircraft was the first option considered for wake alleviation because excess turbulence in the ambient fluid was known to disrupt or prevent the formation of laboratory vortices. Therefore, when the FAA/NASA wake vortex program was started
in the early 1970s, turbulence injection (or addition of turbulence to the wake by means of various devices) was the first method tried.

Application of some of the other concepts could require extensive modification of the wake generating aircraft, which would reduce the attractiveness of the wake alleviation concept. The use of span loading on the wake-generating wing and of wing fins to de-intensify the vortices are then considered. Various wake-vortex dispersion mechanisms have then been developed, which included, among others: spoiler and spline devices, wingtip blowing, forward or reverse thrust of the engines, flight spoilers already on aircraft, wingtip turbine, span loading, wing fins, and passive control.

Mathematical development is discussed only in fundamental forms to allow basic understanding of the physical nature of and role played by aircraft trailing vortices. The dynamics of the vortices in their various configurations are relevant to aircraft flight as lift generating and induced drag generating mechanism, as well as their impact other vehicles and environment. Increasing air traffic density and the introduction of even larger aircrafts have enhanced research and development activities devoted to wake vortex dynamics and safety issues.

The primary purpose of research in this area is to find a way to reduce the hazard potential of lift-generated vortices shed by subsonic transport aircraft in the vicinity of airports during landing and takeoff operations, as schematically illustrated in Figures 4 and 5. The in-trail spacing of the wakes of a main aircraft, poses hazards on other aircrafts during take-off and landing of the main aircraft, since the trailing vortices of the main aircraft could induce a vertical-load and rolling moment dangerous to other aircrafts in the main aircraft’s vicinity in carrying out their maneuvers otherwise. Studies on the dynamics, structure, controllability and management of trailing vortices can perhaps be best analysed by observing the phenomena exhibited by many aircrafts in flight. Figure 6, as another pictorial synopsis of the wake vortex issue, exhibits the trailing edge vortices in the NASA trailing wake vortex study as water entrained in cores makes visible the vortices in the far wake of an aircraft flying at cruise altitude.

In this conjunction, as illustrated in Figure 7, a series of photographs of the far wake of an aircraft flying at cruise altitude have been taken at ground level by Bristol et al [9]. As discussed by Bristol et al. [9], the figure demonstrates simultaneous long- and short-wave instabilities on trailing vortices being studies by aerodynamicists.
Two methodologies have been proposed to reduce the likelihood of catastrophic control loss while allowing reduced separation distances, thereby enhancing the efficiency of impacted airports (McCauley et al [10]). One methodology incorporates the evolution of the wake vortices induced by a leading aircraft in order that the trailing aircraft can successfully navigate around the vortical structures of the wake. However, atmospheric winds, which can change direction and speed rapidly, may reposition the wake vortices sporadically, leading to wake encounters for aircraft which were previously clear of the hazard. In addition, measurement limitations may restrict its practical application. The second methodology attempts to modify the wake of the leading aircraft to reduce its coherence downstream and thereby reduce its impact on following aircraft. A few approaches have been successful in producing pronounced acceleration of the wake vortex breakdown. Thus far, the methods with the greatest promise for achieving accelerated incoherence in the trailing wake rely upon inciting Crow-like instabilities in the wake rather than allowing these perturbations to form over the natural time scale. The excitation of Crow-like instabilities to force wake breakdown may be further subcategorized into active and passive methods. Active techniques typically rely upon oscillation of the wing loading to introduce Crow-like perturbations into the wake. These oscillations may result from the deployment of wing flaps, movement of control surfaces, and flexure of the wing surface in a periodic manner. Due to their reliance on physical oscillations that may ultimately lead to structural fatigue, the implementation of active schemes may be hindered by regulatory commissions or maintenance requirements. In contrast, passive techniques rely upon a fixed wing loading to induce the requisite instabilities. These methods often incorporate the wing into the design process, reshaping it to create the vortex structure necessary to incite Crow-like perturbations. The extension of such technique to six- or eight-vortex systems may allow notable structural or aerodynamic penalties to be redistributed or reduced. Passive techniques may then offer a built-in solution for future aircraft without the need for active control mechanisms.

**Modeling of Trailing Vortices for Passive Wake Alleviation Analysis.** Based on a basic and simple approach for a limited objective using desktop computer for educational purposes, in the review, simulation and analysis attention is given to address the Passive Wake Alleviation Scheme. To this end Biot – Savart Law and Vortex Lattice Method (VLM) van be employed to calculate
velocity field and visualization. Liu’s [6] work on attempt to use the basics principles to study the flow situation in the near wake and far wake, by using Biot – Savart and different vortex models, and to visualize and compare the results with more sophisticated numerical and experimental results. McCauley et al [10] passive alleviation scheme can be followed to further investigate the influence of various distribution of vortices produced by passive wake alleviation scheme on the rolling moment on the following aircraft. For this purpose, the wing set up used by McCauley, as exhibited in Figure 8, will also be reviewed. Outboard triangular flaps for passively inciting Crow-like instabilities in the trailing vortices will be considered. The review, simulation studies as appropriate and analysis thus carried will be limited to qualitative analysis of the general wake features.

A four-vortex wake generated by rectangular wings with outboard triangular has been suggested as an effective passive scheme for wake alleviation [10]. Concurrent numerical studies by Bristol et al., confirmed the qualitative results observed during the experiments of the triangular-flapped wing [9]. More recently, Winckelmans et al. found similar qualitative results using a hybrid vortex method [11].

To further investigate the four-vortex wake flow and in preparation for extensions to six- or eight-vortex systems, McCauley [10] carried out a numerical study of the wake alleviation aspects of the vortex system. The problem as outlined in Figure 8 shows sketches of the two-vortex wake behind a rectangular wing (left column) and the four-vortex wake behind an outboard triangular-flapped wing (right column). The wing planforms are shown in (c). Lift distributions (a) and their derivatives (b) are calculated from a finite wing analysis for planar wings, hence the curvature effects of the wings used in experiments are not included. Vortex sheet roll up is sketched in (d). The far-field wake vortex structure is shown in (e). The vorticity centroids for each half of the wings are marked McCauley et al [10].

Figure 8: Sketches of the vortex wake evolution behind a rectangular (left column) and an outboard triangular flapped (right column) wing planforms shown in (c). Lift distributions are shown in (a) and their derivatives in (b). Vortex sheet roll up is sketched in (d). The far-field wake vortex structure is shown in (e). The vorticity centroids for each half of the wings are marked McCauley et al [10].

Biot – Savart Law. The two-dimensional (point) vortex singularity is one of the solutions for Laplace’s equation. The vortex flow is illustrated in Figure 9. The induced tangential velocity by a vortex is defined below:

\[ V_{\theta} = \frac{\Gamma}{2\pi r} \]

where \( \Gamma \) is the field circulation strength and it is constant around the circle of radius \( r \). Because the circulation has the same sign as the vorticity so it is positive in the clockwise direction. Here \( r \) is the radius to flow center.
Throughout this work the circulation is frequently normalized by the root circulation $\Gamma_v$ of the lift producing wing.

\[
G(r) = \frac{\Gamma(r)}{\Gamma_v} = \frac{2\pi V_0(r)}{\Gamma_v}
\]

the root circulation based on the elliptical loaded wing is given by:

\[
\Gamma_v = \frac{Mg}{\rho s_0 b V}
\]

with $Mg$ as the weight of the aircraft, $b$ as wing span, $V$ as the free stream velocity, $s_0$ as the spanwise load factor. For elliptically loaded wing, $s_0$ is equal to $\pi/4$.

**Horseshoe Vortex.** A special form of vortex which is used in the vortex lattice method is the horseshoe vortex. In this case the vortex line is assumed to be placed in the x-y plane as shown in Figure 9. The horseshoe is actually a simplified case of the vortex ring. It consists of four vortex filaments.

The two trailing vortex segments AB and CD are placed parallel to the x axis and start in infinity. Two finite vortex segments BC and AD. Normally the effects of AD can be neglected because of the infinite distance, in practice the horseshoe vortex contains three parts. The straight bound vortex segment BC models the lifting properties and the two semi-infinite trailing vortex lines model the wake. So the general expression of the induced velocity at a point by the horseshoe vortex is:

\[
V = V_{BC} + V_{B\infty} + V_{C\infty}.
\]

For the finite length vortex segment BC in the horseshoe vortex, the induced velocity at certain point can be calculated using the following equation where $r_1$ and $r_2$ are the distances from this certain point to the two end points of the segment. $r_0$ is the length of the segment.

\[
V_p = \frac{\Gamma}{4\pi} \left[ \frac{r_1 \times r_2}{|r_1 \times r_2|^2} \right] \left[ r_0 \left( \frac{1}{|r_1|} - \frac{1}{|r_2|} \right) \right].
\]

![Figure 9: (a) Nomenclature used for a horseshoe vortex element [12, 13]; (b) The horseshoe vortex lattice model for the classical vortex lattice method, adapted from [6, 12].](image)

**Vortex Models and Vorticity Distribution.** Four different vortex profiles will be compared throughout their properties. The maximum induced rolling moment depends on the vortex model, the core radius and the vortex spacing. A short distance behind the wing of an aircraft the cross-flow kinetic energy of the vortex pair is directly related to the induced drag of the wing and this provides a condition for the initial vortex core size.

6
The four vortex models considered are the Rankine, Lamb – Oseen, Winckelmans and Jacquin vortex models. The Rankine vortex is the result of a discontinuous combination of two solutions of the 2D anti-symmetrical vorticity equation with circular streamlines. It is combined of the pure rotational region and a pure circulation region. The inner part of the vortex is in solid rotation, then its modulus is proportional to r, while the outer part is inversely proportional to the radial distance r. The velocity field calculation is carried out using in-house MATLAB® program. Earlier in-house developed program produced wake geometry result [13] is exhibited in Figure 10.

The decay of the vortex core radius can be calculated with an effective viscosity turbulence model; the decay of the wake vortex circulation and decay of induced rolling moment can be obtained.

**Model 1: Rankine vortex.** The Rankine vortex is the result of a discontinuous combination of two solutions of the 2D antisymmetrical vorticity equation with circular streamlines. It is combined of the pure rotational region and a pure circulation region. The inner part of the vortex is in solid rotation, then its modulus is proportional to r, while the outer part is inversely proportional to the radial distance r. The maximum intensity of the flow is reached at the characteristic distance of the vortex, where there is the change between the inner linear behavior and the external hyperbolic one. The velocity profile is defined as:

\[
V_\theta(r) = \begin{cases} 
\frac{\Gamma_v}{2\pi r_c} \left( \frac{r}{r_c} \right) & -r_c \leq r \leq r_c \\
\frac{\Gamma_v}{2\pi} & \text{otherwise}
\end{cases}
\]

**Model 2: Lamb – Oseen vortex.** The Lamb – Oseen vortex model is a solution to the one-dimensional laminar Navier – Stokes equations, i.e. an antisymmetric solution for the swirl velocity with the assumption that the axial (streamwise) and radial velocities are zero. It is the result of normalizing a Gaussian vortex in such a way that the peak velocity occurs at the core radius. The velocity profile is defined as:

\[
V_\theta(r) = \frac{\Gamma_v}{2\pi} \left[ 1 - e^{-\left( \frac{r}{r_c} \right)^2} \right]
\]

with \(a = 1.256431\). This factor is merely used to put the peak of velocity at the core radius.

**Model 3: Winckelmans.** This is a high order algebraic model proposed by Winckelmans. The velocity profile is defined as:

\[
\]
\[ V_\theta (r) = \frac{\Gamma c}{2\pi r} \left[ \frac{r^2 \left( r^2 + 2\gamma \xi^2 \right)}{(r^2 + 2\gamma \xi^2)^2} \right] \]

with \( \gamma = \frac{\sqrt{17} + 3}{4} \).

**Simulation of Various Vortex Models.** To gain some insight into the influence of a distribution of vortices on the downwash distribution in the flow field, a simulation of a pair of counter rotating Rankine, Lamb – Oseen and Winckelmans Vortices is carried out and the resulting velocity distribution is depicted in Figure 11a, b, and c, respectively.

Figure 11a: Induced velocity distribution by a pair of Rankine Vortices

Figure 11b: Induced velocity distribution by a pair of Lamb – Oseen Vortices

Figure 11c: Induced velocity distribution by a pair of Winckelmans Vortices

Further development can be carried out to incorporate these vortices in vortex filament illustrated. Such procedure is carried out by Liu [6], and some of the results are exhibited in Figures 12, 13 and 14. The maximum intensity of the flow is reached at the characteristic distance of the vortex. The Lamb – Oseen vortex model is a solution to the one-dimensional laminar Navier – Stokes equations (Liu [6]), which is the result of normalizing a Gaussian vortex in such a way that the peak velocity occurs at the core radius.
Figure 12: Normalised Tangential velocity profiles (a) five classic models compared with DG1, (b) four DG cases (DG1-DG4) [6].

Figure 13: Circulation profiles: (a) five classic models compared with DG1, (b) four DG cases (DG1-DG4) [6].

Figure 14: Vorticity profiles of various models for the same circulation strength, normalized by Rankine model. (a) five classic models compared with DG1, (b) four DG cases (DG1-DG4) [6].

**Vorticity Distribution.** Following McCauley, enstrophy\(^2\) will be used to describe the distribution of vorticity in the wake, when the vorticity is not all of the same sign. When all three components of the vector are available as in the case of the CFD calculations, \(\omega^2 = \omega_x^2 + \omega_y^2 + \omega_z^2\) will be used to indicate the complete definition of enstrophy; \(\omega_z^2\) will be used for the vorticity along the z-axis when data is available in one plane only. The total enstrophy \(EN\) is determined as

\[
EN = \int \omega^2 \, dA
\]

and centroid position \(x_{EN} = (x_{EN}, y_{EN})\) is given by

\[
x_{EN} = \frac{1}{EN} \int \int x \omega^2 \, dA = \frac{1}{EN} \int \int x \omega_z^2 \, dA = \frac{1}{EN} \int \int \omega_z^2 \, dA.
\]

For an equi-strength counter-rotating vortex pair, the enstrophy centroid is at the midpoint of the vortices while the vorticity centroid is undefined. The enstrophy dispersion radius \(r_{EN}\) is defined as

---

1 DG – Double Gaussian Model. Further information is given by Li [6]. These velocities are normalised by Rankine model.

2 In fluid dynamics, the *enstrophy* can be interpreted as another type of potential density (i.e. see probability density); or, more concretely, the quantity directly related to the kinetic energy in the flow model that corresponds to dissipation effects in the fluid.
\[ r_{EN}^2 = \frac{1}{EN} \int |x - x_{EN}|^2 \omega^2 dA \]

to describe the extent of the spread of vorticity in the flow. Similar calculations are carried out for \( EN_z \) based on \( \omega_z^2 \).

**Downwash and Rolling Moment.** Forces and moments experienced by a following wing can be used as an effective measure of wake vortex hazard. The experiments for such approach can be further simplified by using the velocity induced by a generating airfoil to deduce the forces experienced by a smaller trailing wing (Rossow, [15, 16]. Following McCauley et al [10], the lift experienced by a following rectangular wing of span \( b_f \) is

\[ L_y = \frac{1}{2} C_{Lo} \rho U_0^2 c_f \int_{-b_f/2}^{b_f/2} \int_{-b_f/2}^{b_f/2} \frac{v(x',y')}{U_0} dx'. \]

The trailing aircraft speed \( U_0 \) will be assumed to be equal to the generating aircraft speed \( U \) and the average integral quantity

\[ D = -\frac{1}{b_f} \int_{-b_f/2}^{b_f/2} \int_{-b_f/2}^{b_f/2} v(x',y') dx' \]

will be used as a substitute for the downwash. The induced rolling moment is

\[ M_y = \frac{1}{2} C_{Mo} \rho U_0^2 c_f \int_{-b_f/2}^{b_f/2} \int_{-b_f/2}^{b_f/2} x' \frac{v(x',y')}{U_0} dx' \]

and the average integral quantity

\[ R = -\frac{1}{b_f} \int_{-b_f/2}^{b_f/2} \int_{-b_f/2}^{b_f/2} x' v(x',y') dx' \]

will be considered.

Excerpts of some basic results obtained by Liu and McCauley et al are reproduced below to exemplify the promise of the approach. Figure 15 exhibits the vorticity, induced downwash, and rolling moment of a Lamb – Oseen vortex. Numerical and analytical calculations are indistinguishable. The extreme values for downwash and rolling moment are each indicated by an *[10]. Figure 16 illustrates the induced rolling moment coefficient on wing around a A380 leader aircraft followed by an A320 obtained by Li [6] using such approach.

![Figure 15: Vorticity (left), induced downwash (center), and rolling moment (right) of a Lamb – Oseen vortex. Numerical and analytical calculations are indistinguishable (adapted from McCauley et al. [10]).](image-url)
Conclusion. Discussion presented here outlines the aircraft trailing vortex hazard alleviation by passive means. For this purpose, simulation studies to explore such possibilities are elaborated following a fundamental and generic approach, which could be carried out meticulously to provide insight as well as applicable solutions. For the computation of the wake vortex encounter problem, different vortex profiles can be compared with respect to their vortex parameters. The present simulation, review and analysis focus on the induced rolling moment coefficient by wakes composed of analytically defined vortex pairs from a leader aircraft. At short distance behind an aircraft, the cross-flow kinetic energy per unit length of the wake is within good approximation equal to the induced drag. This provides a constraint for minimum core radius for the different analytical models. Assuming high Reynolds number similarity, the shape of the velocity profile does not change but the vortex core radius grows slowly downstream. With an initial core radius defined, and a closed expression for the local decay rate of cross-flow kinetic energy given from the theoretical model, a model of vortex core growth for laminar wakes is obtained.

Wakes of practical interest may however display a faster vortex core radius growth, because the flow around the vortex core is no longer laminar. The spectral CFD study presented by McCauley et al here accurately predicts integral properties of a four-vortex wake proposed as solution for wake alleviation. The rolling moment and downwash fields are similar in both numerical and experimental studies and the trend of accelerated wake alleviation occurs in a similar manner in both studies.

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